

The Scalar Quarkonium Spectrum and Quarkonium-Glueball Mixing

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We evaluate the valence approximation to the mass of scalar quarkonium and to the mixing energy between scalar quarkonium and the lightest scalar glueball for a range of different lattice sizes and quark masses. Our results support the identification of $f_0(1710)$ as the lightest scalar glueball.

1. Introduction

The identification of $f_0(1710)$ as the lightest scalar glueball is supported at present by two different sets of calculations. A valence approximation calculation on GF11 [1] of the couplings for the lightest scalar glueball to all possible pseudoscalar pairs, using a $16^3 \times 24$ lattice at $\beta = 5.7$, gives 108 ± 29 MeV for the total two-body width. Based on this number, any reasonable guesses for the effect of finite lattice spacing, finite lattice volume, and the remaining width to multibody states yield a total width small enough for the lightest scalar glueball to be seen easily in experiment. For the valence approximation to the infinite volume continuum limit of the lightest scalar glueball mass, a reanalysis [2] of a calculation on GF11 [3], using 25000 to 30000 gauge configurations, gives a mass of 1710(63) MeV. An independent calculation by the UKQCD-Wuppertal [4] collaboration, using 1000 to 3000 gauge configurations, when extrapolated to the continuum limit according to Ref. [5] gives a mass of 1625(92) MeV. The two results combined give 1693(60) MeV.

Among established resonances with the quantum numbers to be a scalar glueball, aside from $f_J(1710)$ all are clearly inconsistent with the mass calculations expect $f_0(1500)$. Refs. [1,8] interpret $f_0(1500)$ as dominantly composed of $s\bar{s}$ scalar quarkonium. A problem with this interpretation, however, is that $f_0(1500)$ does not seem to decay mainly into states containing an s and an \bar{s} quark [6]. In part for this reason, Ref. [7] interprets $f_0(1500)$ as the lightest scalar glueball

and $f_0(1710)$ as $s\bar{s}$ scalar quarkonium. A second problem is that the Hamiltonian of full QCD couples quarkonium and glueballs so that $f_J(1710)$ and $f_0(1500)$ could both be linear combinations of quarkonium and a glueball, perhaps even half glueball and half quarkonium each.

For several different values of lattice spacing, lattice volume and quark mass, we have now calculated the valence approximation to the mass of the lightest scalar $q\bar{q}$ state and the mixing energy between these states and the lightest scalar glueball. The infinite volume continuum limit of the mass of the scalar $s\bar{s}$ state we find lies at least 200 MeV below the mass of the lightest scalar glueball, ruling out the interpretation of $f_0(1710)$ as an $s\bar{s}$ state with $f_0(1500)$ as a glueball. For the mixing energy evaluated at the strange quark mass, $E(m_s)$, and at the average light quark mass, $E((m_u + m_d)/2)$, we find the infinite volume continuum limit of $E((m_u + m_d)/2)/E(m_s)$ to be 1.31(12). If this ratio is placed in the model of Ref. [9] we find that $f_0(1500)$ is at least 75% $s\bar{s}$ quarkonium and that $f_0(1710)$ is at least 75% glueball. In addition, we find that $f_0(1500)$ acquires a significant component of $(u\bar{u} + d\bar{d})/\sqrt{2}$ from mixing with $f_0(1390)$, composed mainly of $(u\bar{u} + d\bar{d})/\sqrt{2}$. The $s\bar{s}$ and $(u\bar{u} + d\bar{d})/\sqrt{2}$ components of $f_0(1500)$, however, have opposite sign, interfere destructively in decay to $K\bar{K}$ final states and suppress the partial width of $f_0(1500)$ by a factor of 0.5 or less in comparison to an unmixed $s\bar{s}$ state. This suppression provides an explanation for the small observed $K\bar{K}$ partial width.

Our calculations were done on ensembles of 2288 configurations on $16^2 \times 14 \times 20$ at β of 5.93, 1781 configurations on 24^4 at β of 5.93, 582 con-

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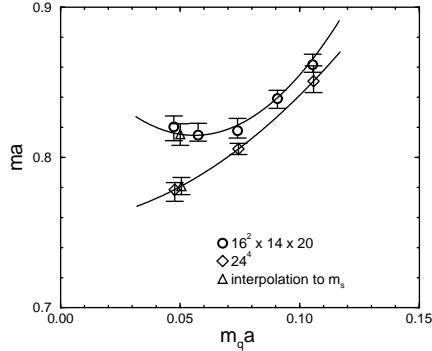


Figure 1. Scalar quarkonium mass as a function of quark mass for β of 5.93

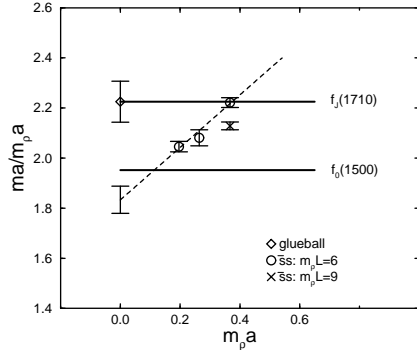


Figure 2. Continuum limit of the scalar $s\bar{s}$ mass.

figurations on $24^2 \times 20 \times 32$ at β of 6.17, and 1003 configurations on $32^2 \times 28 \times 40$ at β of 6.40. To facilitate extrapolation to zero lattice spacing, the smaller lattice with β of 5.93, and the lattices with β of 6.17 and 6.40 were chosen to be nearly the same size in units of the rho Compton wavelength, with periods $m_\rho L$ of 6.1(1), 6.6(2) and 6.3(2), respectively [10], in the two equal space directions.

Following Ref [8] we evaluated on each ensemble correlation functions with random sources built from quark and antiquark fields. Averaging over random sources yields the correlation functions

$$\begin{aligned} C_{ff}(t) &= \sum_{\vec{x}} \langle f(\vec{x}, t) f(0, 0) \rangle, \\ C_{g\sigma}(t) &= \sum_{\vec{x}} \langle g(\vec{x}, t) \sigma(0, 0) \rangle, \end{aligned} \quad (1)$$

for f either σ , π or g , where $\sigma(\vec{x}, t)$ and $\pi(\vec{x}, t)$ are, respectively, the smeared scalar and pseudoscalar

operators of Ref. [8] built from a quark and an antiquark field and $g(\vec{x}, t)$ is the smeared scalar glueball operator of Ref. [3]. We omit, for lack of space, a discussion of our choices of smearing parameters.

From the large t asymptotic behavior of the diagonal correlators,

$$C_{ff}(t) \rightarrow Z_f \exp(-m_f t), \quad (2)$$

we found the masses m_σ , m_π , and m_g and field strength renormalization constants Z_σ , Z_π and Z_g . From the asymptotic behavior of the off-diagonal correlator for m_σ close to m_g

$$C_{g\sigma}(t) \rightarrow \sqrt{Z_g Z_\sigma} E \sum_{t'} \exp(-m_\sigma |t - t'| - m_g |t'|) \quad (3)$$

we then found the glueball scalar-quarkonium mixing energy E . Eqs. (2) and (3) have been simplified by omitting terms arising from propagation around the lattice's periodic time boundary.

Figure 1 shows the scalar quarkonium mass as a function of quark mass for the two lattices with β of 5.93. For both lattices the solid lines are fits to quadratic functions of quark mass which we used to interpolate to the strange quark mass. The strange quark mass we chose as the value which yields, in physical units, a pseudoscalar mass squared of $2m_K^2 - m_\pi^2$, for m_K and m_π the observed kaon and pion masses, respectively. As shown by the figure, for the lattice $16^2 \times 14 \times 20$ with $m_\rho L$ of 6.1(1) the scalar mass as a function of quark mass flattens out as quark mass is lowered toward the strange quark mass and then actually begins to rise as the quark mass is decreased still further. This feature is absent from the data for 24^4 with $m_\rho L$ of 9.2(2) at β of 5.93 and is thus a finite-volume artifact. It is present also in the data, not shown, at β of 6.17 with $m_\rho L$ of 6.6(2) and at β of 6.40 with $m_\rho L$ of 6.3(2).

Figure 2 shows a linear extrapolation to zero lattice spacing of the $s\bar{s}$ scalar mass, measured in units of the rho mass, for $m_\rho L$ near 6, along with the point at β of 5.93 with $m_\rho L$ near 9. It is clear that for $m_\rho L$ of 6, the continuum limit of the $s\bar{s}$ scalar mass lies more than 200 MeV below the valence approximation to the scalar glueball mass. Figure 2 shows also that as $m_\rho L$ is

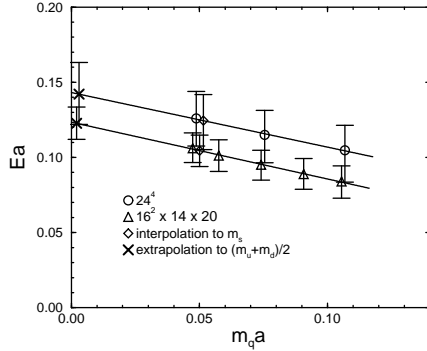


Figure 3. Glueball-quarkonium mixing energy as a function of quark mass for β of 5.93

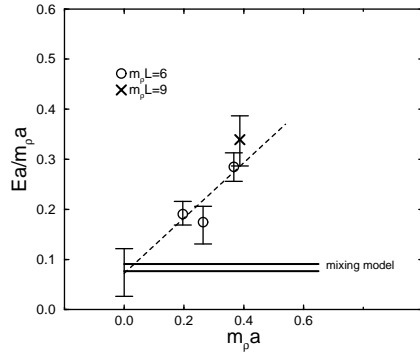


Figure 4. Continuum limit of the glueball-quarkonium mixing energy with $m_\rho L$ of 6.

increased from 6 to 9, the $s\bar{s}$ scalar mass falls. Thus the infinite volume continuum limit of the $s\bar{s}$ scalar mass should be more than 200 MeV below the scalar glueball mass.

Figure 3 is the analogue of Figure 1 for quarkonium-glueball mixing energy. For mixing energy the quark mass dependence shows no anomaly. The mixing energies at different quark masses turn out to be highly correlated and depend quite linearly on quark mass. For β of 6.17 and 6.40 the mixing energy behaves similarly. It appears that the mixing energy can be extrapolated reliably down to the average light quark mass $(m_u + m_d)/2$. For the data at β of 5.93, the mixing energy ratio $E((m_u + m_d)/2)/E(m_s)$ is 1.17(3) for $m_\rho L$ of 6.1(1) and 1.14(5) for $m_\rho L$ of 9.2(2). The ratio has at most rather small volume dependence and thus seems already to be near its infinite volume limit with $m_\rho L$ of 6.1(1).

Figure 4 shows a linear extrapolation to zero

lattice spacing of quarkonium-glueball mixing energy at the strange quark mass giving a limit of 56(37) MeV. A linear extrapolation of the ratio $E((m_u + m_d)/2)/E(m_s)$ gives a continuum limit of 1.31(12).

If our predicted value of the ratio of mixing energies is used in the mixing model of Ref. [9], we obtain the mixing results described earlier. With this choice of mixing energy ratio, an additional prediction of the model is that the state $f_0(1400)$, composed mainly of $(u\bar{u} + d\bar{d})/\sqrt{2}$, picks up most of the glueball amplitude that leaks out of $f_0(1710)$ and therefore should be significantly more prominent in J/Ψ radiative decays than is the $f_0(1500)$. This prediction is consistent with SLAC Mark III data [11]. For the mixing energy evaluated at the strange quark mass, $E(m_s)$, the model predicts 72(6) MeV. The one sigma upper and lower bounds on the model's prediction are shown as horizontal lines in Figure 4 and are consistent with the extrapolated continuum value.

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